TABLET CLASS

Chapter Review

Introduction to Matrices

a matrix is a way to organize information (data) in rows and columns

Temperature for these days

$$\begin{array}{cc} & \begin{array}{ccc} M & W & F \end{array} \\ \begin{array}{c} \text{Hi} \\ \text{Low} \end{array} & \begin{bmatrix} 90 & 92 & 95 \\ 50 & 51 & 53 \end{bmatrix} \end{array}$$

order is the size of a matrix

$$\text{2 Rows} \quad \underbrace{\begin{bmatrix} 90 & 92 & 95 \\ 50 & 51 & 53 \end{bmatrix}}_{\text{3 columns}} \quad \begin{array}{c} \text{order} \\ 2 \times 3 \end{array}$$

Matrix Operations

You can $+, -, \times, \div$ matrices

however, certain conditions apply

Adding / Subtracting Matrices - must have same size.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 9 & -1 \\ 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 11 & 2 \\ 5 & 2 & 14 \end{bmatrix}$$

add respective entries

Scalar multiplication (a number $\times$ matrix)

$$3\begin{bmatrix} 1 & 5 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 15 \\ 18 & 9 \end{bmatrix}$$

multipy 3 to all entries

Matrix Multiplication

First, determine if two matrices can be multiplied

$$(2 \times 3) \times (3 \times 2)$$

numbers are equal; can multiply

product will be a 2×2 matrix

Procedure

use rows

$$\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} R1C1 & R1C2 \\ R2C1 & R2C2 \end{bmatrix}$$

use columns

$$\begin{bmatrix} 1(5) + 3(2) & 1(1) + 3(4) \\ 0(5) + -2(2) & 0(1) + -2(4) \end{bmatrix} = \begin{bmatrix} 11 & 13 \\ -4 & -8 \end{bmatrix}$$

Determinants

A determinant is a number associated with every square matrix - it has many valuable applications

Determinant of a 2×2 matrix

Given $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$ find $\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$

matrix   determinant

find $\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = (5 \cdot 2) - (1 \cdot 3)$

$= 10 - 3$

$= 7$

Determinant of 3×3 matrix - two methods

- Expansion Method
- Diagonals Method

Expansion Method

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 5 & 2 \\ 1 & 0 & 3 \end{bmatrix} \quad \text{find} \quad \overbrace{\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 2 \\ 1 & 0 & 3 \end{vmatrix}}^{\text{use } 2\times 2 \text{ determinants}}$$

Example – expand along row 1 – see pattern

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 2 \\ 1 & 0 & 3 \end{vmatrix} = 1\begin{vmatrix} 5 & 2 \\ 0 & 3 \end{vmatrix} = 1(15-0) = 15$$

$$-\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 2 \\ 1 & 0 & 3 \end{vmatrix} = -2\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = -2(9-2) = -14$$

$$+\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 2 \\ 1 & 0 & 3 \end{vmatrix} = 4\begin{vmatrix} 3 & 5 \\ 1 & 0 \end{vmatrix} = 4(0-5) = -20$$

$$\text{Determinant} = 15 + -14 + -20 = -19$$

Sign pattern along rows

$$\begin{vmatrix}1&2&4\\3&5&2\\1&0&3\end{vmatrix} \quad \begin{matrix}+&-&+\\-&+&-\\+&-&+\end{matrix}$$

$\boxed{\text{try to expand along rows with the most zeros}}$

Diagonal Method

$$\begin{bmatrix}1&2&4\\3&5&2\\1&0&3\end{bmatrix} \qquad \text{find } \begin{vmatrix}1&2&4\\3&5&2\\1&0&3\end{vmatrix}$$

step 1 - rewrite the first 2 columns after

$$\underbrace{\begin{vmatrix}1&2&4\\3&5&2\\1&0&3\end{vmatrix}}_{\text{determinant}}\underbrace{\begin{matrix}1&2\\3&5\\1&0\end{matrix}}_{\text{first 2 columns}} =$$

Second, subtract these products

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 2 \\ 1 & 0 & 3 \end{vmatrix}\begin{matrix} 1 & 2 \\ 3 & 5 \\ 1 & 0 \end{matrix} =$$

First, add these products

$$(\underline{15 + 4 + 0}) - (\underline{20 + 0 + 18}) =$$

$$\underline{19} - (\underline{38}) = -19$$

Identity and Inverse Matrices

$$a \cdot 1 = a$$

1 is the identity

$$7 \cdot 1 = 7$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$$

identity matrix (2×2)

Inverse of a 2 x 2 matrix

Given	Inverse
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	$A^{-1} = \frac{1}{\lvert A \rvert}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

note = $ad - cb \neq 0$

Example

$$A = \begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\begin{vmatrix} 4 & 1 \\ 2 & -2 \end{vmatrix}}\begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix}$$

opposite

FLIP

Determinant

$$A^{-1} = \frac{1}{-8-2}\begin{bmatrix} -2 & -1 \\ -2 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-10}\begin{bmatrix} -2 & -1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 2/10 & 1/10 \\ 2/10 & -4/10 \end{bmatrix}$$

Solving Systems using Inverse Matrices

Formula

$$\boxed{\begin{aligned} A \cdot X &= B \\ X &= A^{-1}B \end{aligned}}$$

$$\begin{cases} 5x + 4y = 6 \\ -2x - 3y = -1 \end{cases}$$

$$\boxed{\begin{bmatrix} 5 & 4 \\ -2 & -3 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$$

Find inverse

$$A = \begin{bmatrix} 5 & 4 \\ -2 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-15+8} \begin{bmatrix} -3 & -4 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = -1/7 \begin{bmatrix} -3 & -4 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3/7 & 4/7 \\ -2/7 & -5/7 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$\underset{(2 \times 2)}{\begin{bmatrix} 3/7 & 4/7 \\ -2/7 & -5/7 \end{bmatrix}} \underset{(2 \times 1)}{\begin{bmatrix} 6 \\ -1 \end{bmatrix}} =$$

$$X = A^{-1} \cdot B$$

$$\underset{(2 \times 2)}{\begin{bmatrix} 3/7 & 4/7 \\ -2/7 & -5/7 \end{bmatrix}} \underset{(2 \times 1)}{\begin{bmatrix} 6 \\ -1 \end{bmatrix}} = \overset{(2 \times 1)}{\begin{bmatrix} (18/7 + -4/7) \\ (-12/7 + 5/7) \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14/7 \\ -7/7 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad (2,-1)$$

solution

Solving Systems using Cramer's Rule

Another way to solve systems using determinants – the formulas look complex but the rule is easy to use – see example

$$\begin{Bmatrix} ax + by = c \\ dx + ey = f \end{Bmatrix}$$

2 x 2 system

$$x = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

3 x 3 system

$$\begin{Bmatrix} ax + by + cz = d \\ ex + fy + gz = h \\ ix + jy + kz = l \end{Bmatrix}$$

$$x = \frac{\begin{vmatrix} d & b & c \\ h & f & g \\ l & j & k \end{vmatrix}}{\begin{vmatrix} a & b & c \\ e & f & g \\ i & j & k \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a & d & c \\ e & h & g \\ i & l & k \end{vmatrix}}{\begin{vmatrix} a & b & c \\ e & f & g \\ i & j & k \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} a & b & d \\ e & f & h \\ i & j & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ e & f & g \\ i & j & k \end{vmatrix}}$$

Solving systems using ***Cramer's Rule***

$$\begin{cases} -2x + y = 8 \\ 3x + y = -2 \end{cases} \quad (-2,4)$$

notice the x column was replaced

$$x = \frac{\begin{vmatrix} 8 & 1 \\ -2 & 1 \end{vmatrix}}{\begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} -2 & 8 \\ 3 & -2 \end{vmatrix}}{\begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix}}$$

notice the y column was replaced

the coefficient of the x and y columns

Calculate the determinants and simplify

$$x = \frac{10}{-5} \qquad y = \frac{-20}{-5}$$

$$x = -2 \qquad y = 4$$

$$(-2,4)$$

$$\begin{cases} 3x + 2y + 4z = 11 \\ 2x - y + 3z = 4 \\ 5x - 3y + 5z = -1 \end{cases}$$

$$\begin{vmatrix} 3 & 2 & 4 \\ 2 & -1 & 3 \\ 5 & -3 & 5 \end{vmatrix} = 18$$

$$x = \frac{\begin{vmatrix} 11 & 2 & 4 \\ 4 & -1 & 3 \\ -1 & -3 & 5 \end{vmatrix}}{18} = \frac{-54}{18} = -3$$

$$y = \frac{\begin{vmatrix} 3 & 11 & 4 \\ 2 & 4 & 3 \\ 5 & -1 & 5 \end{vmatrix}}{18} = \frac{36}{18} = 2$$

$$z = \frac{\begin{vmatrix} 3 & 2 & 11 \\ 2 & -1 & 4 \\ 5 & -3 & -1 \end{vmatrix}}{18} = \frac{72}{18} = 4$$

solution $(-3, 2, 4)$